# Period in the chaotic phase of Q2R automata



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**LETTER TO THE EDITOR**

# Period in the chaotic phase of Q2R automata

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**Abstract.** More than 1000 h on a high-speed special computer were used to analyse the global limit cycle period of the Q2R automata in 2D at various energies. An exponential increase of the periods with system size $N$ was found even at energies below the Curie temperature. Even for $T \to \infty$, i.e. in the fully chaotic regime, the paramagnetic phase of Q2R was found to be non-ergodic.

The crucial question whether cellular automata, particularly the Q2R automata, can simulate the Ising magnet [1] is not yet completely answered. For low energies, constant oscillations in the magnetisation of the Q2R automata have been observed [2]. This could cause incorrect averages. Therefore, we have concentrated on the question of how the 'global limit cycle', i.e. the global period [3] of the Q2R automata depends on the system size for various energies in 2D.

Since a lattice with $N$ spins can only hold $2^N$ different configurations (exactly two per lattice site: 'spin up' or 'spin down'), the system will return to its initial configuration after at most $2^N$ timesteps. During one timestep every spin of the whole lattice is updated. This definition is important, because the square lattice has to be divided into two sublattices, which are sequentially updated.

The intial configuration was determined randomly with a given concentration $p$ (probability) for 'spin up', which represented the energy. In practice we calculated the total number of 'spin ups' for the given concentration and system size and decided randomly their position in the lattice. We found that this method of a fixed concentration $p$ gives the same results but less fluctuations in less computer time compared with the method of deciding for each lattice site its spin direction with a given probability.

Since our problem is quite time consuming even for small lattices, we simulated most of the systems on a K2 processor running at a clock frequency of 30 MHz [4]. We reached 2 million spin flips per second, programming directly in micro-code; two days were needed to find our longest period of nearly 928 million.

For $p = 0.5$, i.e. $T \to \infty$, we found the expected exponential increase of the global period with the system size $N$ (see figure 1). We determined $\lambda$ in $\tau \sim 2^{\lambda N}$ at $\lambda \approx 0.27$. Systems at $T = T_c$ (Curie temperature) seem to have the same behaviour even though the increase is definitely not that strong (figure 2). Similar behaviour was found at a concentration $p = \frac{1}{16}$ of initial 'spin ups', slightly below Curie temperature (figure 2). We conclude that the Curie temperature is not the critical point, below which a different behaviour of the global period might occur. On the other hand, the total number of states at a fixed energy corresponding to $p = 0.5$ (infinite temperatures) most likely varies asymptotically as $2^N$. Thus even in the fully chaotic regime near $p = 0.5$, the

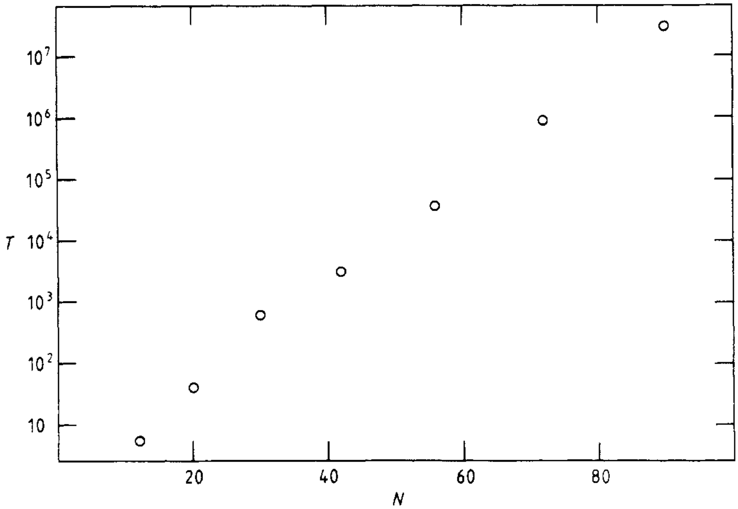


**Figure 1.** Logarithmically averaged $\exp(\langle \ln \tau \rangle)$ global periods, $T$ (circles), at $T = \infty$ ($p = 0.5$). The medians were nearly the same. 50 runs were made for each point.

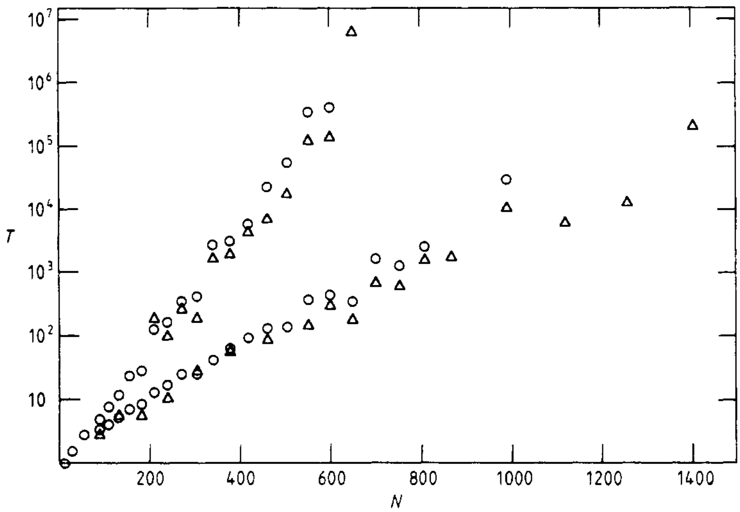


**Figure 2.** Median (triangles) and logarithmically averaged (circles) global periods for $p = p_c = 0.079\,55$ (upper curve) and $p = \frac{1}{16}$ (lower curve). For smaller lattices at $p = \frac{1}{16}$, 500 runs were made. Up to lattices with $N = 200$, median and logarithmic averages for $p = p_c$ are nearly the same.

Q2R algorithm reaches only an exponentially small fraction $\sim 2^{-0.73N}$ of all possible states. In other words, Q2R is not fully ergodic. Earlier work showed [3,5] already for lower temperatures, $p < 0.08$, some deviations from normal Ising models. To our knowledge the non-ergodicity at $p = 0.5$ is the first deviation in the paramagnetic phase of Q2R.

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